Solving the Unsolvable

Conquering gigantic optimization problems with FICO™ Xpress Optimization Suite

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Summary

Solving large, complex optimization problems can be a daunting task. Conquering them effectively, however, can be the difference between success and failure in today’s highly competitive marketplace. For example, retail organizations can efficiently manage floor space, energy companies can optimize production scheduling, marketers can effectively target the right audience with the right offers, personnel planners can generate equitable staff schedules, and transport enterprises can ensure reliable delivery at the least cost.

As problems grow in size and scope, it becomes increasingly difficult to get answers in a timely manner. Recent improvements in optimization solver engine performance have not been sufficient to deal with the challenge of these increasingly complex problems. In many cases, optimization problems are too large to fit into memory, require too much time to compute or are simply too hard to solve.

For these reasons, simply boosting the computational horsepower is often not enough. As one of the ways to address the scalability issue, next-generation solver engines must look for ways to decompose problems into smaller, more manageable components. Doing so will allow even the most complex problems to be solved so that key tactical, strategic and operational decisions can still be made with confidence. Organizations that solve problems efficiently at all levels of complexity have a unique competitive advantage.
Historically, 16- and 32-bit architectures imposed strict limitations on the size of the problem to be crunched without decomposition. Optimization problems that had 1 million variables and 1 million constraints were on the periphery of solvable sizes. Introduction of 64-bit architectures expanded the spectrum of problem sizes and has made it possible to dramatically increase the size of solvable problems. However, on one hand, some super-large-scale problems are still beyond the hardware capability, and on the other hand, the solver performance decays non-linearly with the growth of the problem size. Therefore, large-scale optimization introduces an added level of complexity that requires creativity to ensure a solution to the problem can actually be derived within the acceptable time limit.

Modeling optimization problems is usually an intellectually challenging exercise. Depending on the problem being solved and the number of constraints that need to be considered, the process for creating an adequate model can take a considerable amount of time without the right set of tools. The situation becomes further complicated once a valid model is developed. You are then faced with deciding how to integrate the optimization execution software that utilizes the designed model with a computer application or a suite of products.

To make this process easier, feature-rich solver engines such as FICO™ Xpress Optimizer have been developed to help streamline the task of creating and deploying models for optimization problems. Leveraging a product like Xpress Optimizer, companies can quickly model and implement complex decision making processes across the organization. Increasingly, however, problems are growing in both complexity and size, so traditional solver engines will no longer suffice. With large-scale optimization, while performance still remains a key concern, new methods must be introduced to enable solving of problems that cannot otherwise be solved. To this end, the FICO Xpress Optimization Suite has introduced new capabilities that make large-scale optimization problems more manageable and help turn the unsolvable into solvable. The key element in making this happen is decomposition.

Decomposition is simply the process of breaking large optimization problems into smaller, more manageable sub-problems and solving them either sequentially or in parallel. There are two well-known methods to decomposition, namely Benders, a row generation approach, and Dantzig-Wolfe, a column generation approach.

The solvers and modeling tools of FICO Xpress Optimization Suite support the implementation of decomposition approaches in various ways. Most importantly, the unique features of Xpress-Mosel, the modeling language in the Xpress Optimization Suite, make it a particularly suitable platform for the development of decomposition.

**Decomposition is the process of breaking large optimization problems into smaller, more manageable sub-problems and solving them either sequentially or in parallel.**
Decomposition with Xpress-Mosel

Xpress-Mosel is a high-level modeling language combined with standard functionality of programming languages allowing for the implementation of models and solution algorithms in a single environment. Through its open, modular architecture, extensions to the language can be made without any need for modifications to the core system. The platform-independent compiled models (BIM) are portable across all platforms supported by Xpress and protect your intellectual property on deployment. Various library interfaces are available for embedding models into applications (C, Java, C#, VB). Model development and analysis is supported by the visual development environment Xpress-IVE and a set of tools (debugger, profiler).

Multi-Solver

Xpress-Mosel is designed with an open, modular architecture. Solvers are modules that are loaded into a model as needed. The Xpress suite comes with a comprehensive set of optimization solvers from which you can choose the one that is best suited for your problem type, or you can use several solvers in combination within a single model. Other modules—for example, providing data interfaces or graphics—are also available, and users can even write their own modules to enhance the Mosel language according to their needs.

Multi-Problem

With Xpress-Mosel, multiple optimization problems can be defined within a single optimization model, making the process of handling large-scale optimization problems much easier. At any point a single problem is active, it is possible to switch back and forth between various other problems, allowing for the retrieval of solution information across problem components. Problems can share data, make use of common decision variables and easily copy constraints from one problem to another or duplicate a problem altogether.

Multi-Model

Xpress-Mosel also allows multiple optimization problems to be implemented as separate model (files). This approach is most suitable if the optimization process should be spread along several threads and executed in parallel. Xpress-Mosel’s unique implementation characteristics make parallel and multithreaded optimization easily accessible. Readily available communication mechanisms include synchronization of concurrent models based on event queues and data exchange through shared memory.

Multi-Node

Release 7.1 of Xpress extends Mosel’s capacities for handling multiple models to distributed computing using several Mosel instances (running locally or on remote nodes connected through a network). This facility opens new perspectives for the implementation of decomposition approaches, using all the computing power available in your local network. Moving from a multi-model application on a single instance to a multi-node application only requires few changes in a model, largely due to the concept of I/O drivers—prefixes to the file name that indicate how to access a given file (remotely, in memory, compressed, etc.).

Schemes of decomposition and concurrent solving that can be implemented with Xpress-Mosel include:

- Simple parallel runs (different data instances; different algorithm configurations).
- Decomposition approaches (Benders; Dantzig-Wolfe).
- Column generation (loop over top node; branch-and-price).
- Cut generation (cut-and-branch; branch-and-cut).
Problem solving approaches that involve parallel execution of (sub)models can only be implemented as multiple models, whereas sequential solving can be formulated either way. For sequential algorithms, the developer may choose between the two design options. For more detail on these techniques, refer to the whitepaper "Multiple models and parallel solving with Mosel", located on http://optimization.fico.com.

**Case Study—Retail Space Planning**

**Challenge:**
Retail space planning requires the solution of relatively large Mixed Integer Programs (MIP), very combinatorial in nature, for which the current cutting-edge MIP technology would not find a solution of good quality, even if the solver is left to run for a very long time. Feasibility pump in conjunction with local-search heuristics and other classes of heuristics implemented in Xpress-Optimizer are key techniques of specific broader meta-heuristics designed for retail space optimization. In this case, it is desirable to have Xpress heuristics that can be executed very quickly and to have a modeling environment that gives the user easy and efficient ways to handle multiple large or small models/MIPs in a sequential or parallel fashion.

**Solution:**
The core MIP models for space planning are very large and combinatorial in nature. Xpress-Mosel has been a key choice in the implementation of space optimization for a top US retailer, as it gives efficient ways to handle multiple models and fast interfaces between them (e.g., shared memory). As an example, Mosel can have sub-models running meta-heuristics in parallel, while the Xpress MIP solver attempts to close the optimality gap for the core model, within the allowed time. As soon as a meta-heuristic finds a better solution (which is frequent in this type of application) then the solution is submitted to the master model that is solving the core model. Mosel also proved to be a good choice with regard to development time, as the analysts/developers could focus more on the mathematical programming aspects and not as much on how to implement complex decompositions, interfaces and synchronization mechanisms to handle multiple models and solvers in the same project.

**Benefit:**
One of the key performance goals was to achieve a good quality solution, between 1% and 2% optimality gap, in a very short time, meaning less than 10 minutes. This could only be achieved with the decomposition approach presented above.

Optimized shelf layouts created by Xpress.
**Challenge:**
Production scheduling requires the solution of very large MIPs. A standard way to solve these problems is to apply chronological decomposition heuristics. As in the previous case, having an effective modeling and development environment is crucial in first finding the required quality solutions and then being able to deliver the model in time for deployment.

**Solution:**
Xpress-Mosel has been used for a large US food/drink manufacturer and has been recently enhanced to support multi-million, more automated production/packaging lines together with the old standard lines. This addition has increased the difficulty of solving the underlying MIPs, which are solved by applying other specific hierarchical decomposition heuristics implemented efficiently and effectively using Mosel. Tuning the Xpress MIP solver for these optimization problems proved to be a crucial step in getting the solve times reduced, as well as in getting the required quality solutions. The Xpress visual environment for Mosel modeling and development, called Xpress-IVE, and the Xpress-Tuner have been proficient tools to perform the tuning step.

**Benefit:**
By using this approach we were able to bring down the solve times to approximately five minutes on average, which fulfilled the customer requirements.
Case Study—Online Advertising

Challenge:
Online advertising allocation and pricing models are frequent in today’s Internet market. High volumes of bids need to be processed within extremely short time spans.

Solution:
The Xpress MIP solver has been selected for several years now by one of the largest software players in the market. The model requires the efficient solution of very large linear programs with both hard and soft constraints that include non-linear terms to handle complex cost and inventory functions. The size of this model is much superior to the size of the models from the previous cases. Therefore, in this case, the interface adopted is Xpress-BCL 64 bit. BCL is an object oriented library interface to Xpress-Optimizer. As a library it requires less overhead in its use than the high-level modeling in Mosel and it grants direct access to advanced functionality of the Optimizer library.

Benefit:
Using very efficient implementations of special ordered sets, Xpress has an effective way to approximate the non-linear functions and hence solves the advertising model within the available time limit.

About the FICO™ Xpress Optimization Suite

The Xpress optimization suite includes a wide range of fast solvers and modeling interfaces. The modeling and solving environment of Xpress-Mosel is an easy-to-learn modeling and programming language that supports your efforts from rapid prototyping to in-depth model development and tuning. It comes with a visual development environment (Xpress-IVE), that makes it easier for developers to get new optimization initiatives to market faster. The suite also benefits from the close link between Xpress-Mosel and the Xpress solvers and is thus the perfect tool for the implementation of decomposition approaches. An alternative modeling interface is the builder component library Xpress-BCL. This object-oriented library adds advanced modeling functionality to your preferred programming language (available in Java, C, C++, .Net).
To get the most out of our software, training courses and consultancy ranging from just a few hours to fully implementing customer specific solutions are offered.

**Learn more about Xpress**

Xpress website: [www.fico.com/xpress](http://www.fico.com/xpress)


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